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Higher Derivative Correction to the Hawking Flux via Trace Anomaly

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Abstract

In this paper we derive Hawking radiation of black holes with higher derivative corrections by the method of trace anomaly. Firstly we derive Hawking radiation for general spherical black holes. We introduce a modified tortoise coordinate to it and find the analyticity of the coordinate. Secondly we apply its method to a black hole with a higher derivative correction and derive the Hawking radiation of it. We found that as an ordinary case the flux depends only on the surface gravity.

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1 Introduction

Up to now, Hawking radiation [8] has been investigated by several methods. Hawking originally described the radiation by observing a gravitational collapse, and Unruh [9] found there is a radiation even if one considers an eternal black hole. Following them, Christensen and Fulling [6] discovered new means which is based on a conformal anomaly near a horizon. Recently, Robinson and Wilczek [2] (a modified discussion is given by [3] and [35]) showed that Hawking radiation can be derived by a gravitational anomaly, and many investigations have been made in various situations([4, 5, 7], [11]...[34]).

In this paper, by CFT method based on a conformal anomaly, we survey the energy momentum tensor in infinity which is originated from Hawking radiation in general spherical symmetric black hole backgrounds. The general backgrounds, especially, contain higher derivative corrected black hole geometries which are important to know the effects from string theory. To study via CFT method, we must modify the definition of the tortoise coordinate in a spherical symmetric black hole which has been defined in the case of the metric having a property that the product of time and radial components is -1. And we shows the new definition of the tortoise coordinate is well-defined to prove the Kruskal metric has no coordinate singularity near the horizon. Requiring there are no divergent physical quantities at the horizon in the Kruskal geometry, we calculate the energy momentum tensor and find the result agrees to that of [10, 33] in which they derive it by gravitational anomaly method. We review and simplify it in Appendix A.

This article is organized as follows.

In section 2, preparing for CFT method to investigate the energy momentum tensor of Hawking radiation, we suggest the new definition of the tortoise coordinate and, in the Kruskal coordinate constructed from it, prove that there is no coordinate singularity near the horizon.

In section 3, applying the modified tortoise coordinate, we calculate the energy momentum tensor of Hawking radiation in general spherical black hole backgrounds.

In section 4, using the result of section 3, we derive the higher derivative correction to Hawking flux in a specific case which action gives a non-trivial property to the metric ($g_{tt}g_{rr} \neq -1$) [1] and which would be the simplest one of the metric having such a property.

In Appendix A, we rederive the result of section 3 to use the gravitational anomaly method.

In Appendix B, we present the evidence of the theory having the nature of CFT near the horizon.

2 Kruskal coordinate of general spherical black holes

In this section, for preparation of the later section we examine the feature of the Kruskal coordinate of the general spherical black holes.

In general D dimensional spherical black holes, the metrics have a form of

$$ds^2 = e^{2A(r)}dt^2 - e^{2B(r)}dr^2 - r^2 d\Omega_{D-2}^2. \tag{2.1}$$

Now we write

$$e^{-2B(r)} = e^{2A(r)} L(r)^2, \tag{2.2}$$

and define $r = r_+$ as the greatest real root of

$$e^{2A(r)} = 0, \tag{2.3}$$

which is the radial coordinate of the horizon. We suppose the invariant volume element is non-singular and non-zero at the horizon and out of the horizon , then $L(r)$ becomes non-singular and non-zero in this region, because we have

$$\sqrt{-g} = L(r)^{-1} r^2 \sin\theta. \tag{2.4}$$

In general $L(r) \neq 1$, though we have $L(r) = 1$ for ordinary Einstein-Hilbert action. So we must modify the tortoise coordinate which is defined in the case of $L(r) = 1$. It is desirable that, at the horizon, the metric has no coordinate singularity when we use the Kruskal coordinate constructed of the modified tortoise coordinate.

Now we define the modified tortoise coordinate r_* as the solution of the differential equation,

$$\frac{dr_*}{dr} = e^{B(r)-A(r)}. \tag{2.5}$$

We will show that the above definition leads to no coordinate singularities in the metric in the representation of the Kruskal coordinate.

Using the Eddington-Finkelstein coordinate $u = t - r_*, v = t + r_*$ the metric (2.1) can be written as

$$ds^2 = e^{2A(r)} du dv - r^2 d\Omega_{D-2}^2. \tag{2.6}$$

To obtain an expression of a surface gravity κ_+ defined by the Killing equation

$$\xi^\mu \nabla_\mu \xi^\nu = \kappa_+ \xi^\nu, \tag{2.7}$$

where ξ^μ is a Killing vector, we rewrite (3.1) in the ingoing Eddington-Finkelstein coordinate (v, r, Ω_{D-2}). Then we have

$$ds^2 = e^{2A(r)} dv^2 - 2e^{A(r)+B(r)} dv dr - r^2 d\Omega_{D-2}^2 \tag{2.8}$$

We find that ξ^v survives and is a constant, and other components vanish. By evaluating (2.7) near the horizon. we obtain

$$\kappa_+ = \frac{1}{2}L(r_+)(e^{2A(r)})'|_{r=r_+}. \tag{2.9}$$

We solve the equation (2.5) near the horizon. To do so we rewrite $e^{2A(r)}$ to

$$e^{2A(r)} = \left(1 - \frac{r_+}{r}\right) g(r), \tag{2.10}$$

where $g(r)$ is a holomorphic and non-zero function near $r = r_+$ (we suppose the black hole is non-extremal). The l.h.s. of (2.5) is

$$e^{B(r)-A(r)} = \left(1 + \frac{r_+}{r - r_+}\right) h(r), \tag{2.11}$$

where $h(r) = L(r)^{-1}g(r)^{-1}$ which is holomorphic at the horizon. Near the horizon, $h(r)$ has a Taylor expansion

$$h(r) = a_0 + a_1(r - r_+) + \frac{1}{2}a_2(r - r_+)^2 + \ldots, \tag{2.12}$$

where $a_0 = h(r_+), a_1 = h'(r_+)$, e.t.c.. So (2.11) is

$$(a_0 + a_1 r_+) + \frac{a_0 r_+}{r - r_+} + (terms\ holomorphic\ at\ r = r_+), \tag{2.13}$$

and can be integrated to

$$r_* = a_0 r_+ \ln\left(\frac{r}{r_+} - 1\right) + p(r), \tag{2.14}$$

where $p(r)$ is a holomorphic function at $r = r_+$. The relation between $a_0 r_+$ and κ_+ is

$$\begin{aligned} a_0 r_+ &= L(r)^{-1}g(r)^{-1}r|_{r=r_+} \\ &= [L(r_+)(e^{2A(r)})'|_{r=r_+}]^{-1} = \frac{1}{2\kappa_+}. \end{aligned} \tag{2.15}$$

Finally we have

$$r_* = \frac{1}{2\kappa_+} \ln\left(\frac{r}{r_+} - 1\right) + p(r). \tag{2.16}$$

This is the similar result of the tortoise coordinate of the black hole obtained from the ordinary Einstein-Hilbert action.

Define the Kruskal coordinate,

$$U = -e^{-\kappa_+ u}, V = e^{\kappa_+ v}, \tag{2.17}$$

using (2.16), the metric (3.1) becomes

$$ds^2 = -\frac{r_+ e^{-p(r)}}{\kappa_+^2 r}dUdV - r^2 d\Omega_{D-2}^2. \tag{2.18}$$

As there is no coordinate singularity at $r \geq r_+$, our definition (2.5) is reasonable.

Using the modified coordinate, we will see the Hawking flux via CFT method and obtain the same result of the gravitational anomaly method.

3 Method of Trace Anomaly of Two dimensional CFT

In [6] it is reported that the Hawking flux of the Schwarzschild black hole is derived via the method of trace anomaly in two dimensional CFT. And the method is applied to many kinds of black holes. For example the Hawking flux of the four dimensional charged black hole without higher derivative correction was calculated in this method [7].

In this section, we apply the method to the general spherical black holes and derive the Hawking flux of them. As shown in Appendix B, near the horizon we obtain the effective two dimensional theory with dilaton, which metric is

$$ds^2 = e^{2A(r)}dt^2 - e^{2B(r)}dr^2 = e^{2A(r)}dudv \tag{3.1}$$

The energy-momentum conservation and the current conservation are

$$\nabla_\mu T^\mu_{\ \nu} = F_{\mu\nu}J^\nu, \tag{3.2}$$

$$\nabla_\mu J^\mu = 0. \tag{3.3}$$

And there is matter fields which contribute to the Hawking radiation, the energy-momentum tensor has the trace anomaly

$$T^\mu_{\ \mu} = \frac{1}{24\pi}R. \tag{3.4}$$

The two dimensional chiral current $J^{5\mu} = \frac{\epsilon^{\mu\nu}}{\sqrt{-g}}J_\nu$ satisfies the anomalous conservation law

$$\nabla_\mu J^{5\mu} = \frac{e^2}{2\pi}\frac{\epsilon^{\mu\nu}}{\sqrt{-g}}F_{\mu\nu}. \tag{3.5}$$

From (3.3) and (3.5), we have

$$\partial_u J_v + \partial_v J_u = 0, \tag{3.6}$$

$$\partial_u J_v - \partial_v J_u = \frac{e^2}{\pi}F_{uv}. \tag{3.7}$$

Under the Lorentz gauge $\nabla^\mu A_\mu = 0$, these equations can be solved as

$$J_u = j(u) + \frac{e^2}{\pi}A_u(r), \tag{3.8}$$

$$J_v = \tilde{j}(v) + \frac{e^2}{\pi}A_v(r), \tag{3.9}$$

where $j(u)$ ($\tilde{j}(v)$) is a holomorphic (an anti-holomorphic) function which we will determine later.

In the Kruskal coordinate (2.17) we have

$$J_U = -\frac{1}{\kappa_+ U}\left[j(u) + \frac{e^2}{\pi}A_u(r)\right]. \tag{3.10}$$

As there is no singularity on the horizon when we use the Kruskal coordinate, we require that J_U has a finite value on the horizon, so we have

$$j(u)|_{r\to r_+} = -\frac{e^2}{\pi}A_u(r_+). \tag{3.11}$$

And we require there is no ingoing mode:

$$\tilde{j}(v)|_{r\to\infty} = 0. \tag{3.12}$$

With these requirements we can calculate the current at infinity

$$J^r = \frac{dr}{dr_*}J^{r_*} \to -\frac{e^2}{\pi}A_u(r_+)\ (as\ r\to\infty). \tag{3.13}$$

Next, we calculate the energy momentum tensor. From (3.2) and (3.4) with the Lorentz gauge $\nabla^\mu A_\mu = 0$, we find

$$T_{uu} = t(u) + \frac{1}{12\pi}(\partial_u^2 A(r) - (\partial_u A(r))^2) + \frac{e^2}{\pi}A_u(r)^2 + 2A_u(r)j(u), \tag{3.14}$$

$$T_{vv} = \tilde{t}(v) + \frac{1}{12\pi}(\partial_v^2 A(r) - (\partial_v A(r))^2) + \frac{e^2}{\pi}A_v(r)^2 + 2A_v(r)j(u), \tag{3.15}$$

here $t(u)$ and $\tilde{t}(v)$ are a holomorphic and an anti-holomorphic function respectively.

As above, we require that $T_{UU} \sim \frac{1}{U^2}T_{uu}$ must be regular on the future horizon. Then we have

$$t(u)|_{r\to r_+} = -\frac{1}{12\pi}(\partial_u^2 A(r) - (\partial_u A(r))^2)|_{r=r_+} + \frac{e^2}{\pi}A_u(r_+)^2 \tag{3.16}$$

$$= \frac{1}{192\pi}[(e^{-2B(r)})'(e^{2A(r)})']_{r=r_+} + \frac{e^2}{\pi}A_u(r_+)^2. \tag{3.17}$$

We also require that there is no ingoing flux from infinity

$$\tilde{t}(v)|_{r\to\infty} = 0. \tag{3.18}$$

Then we obtain the energy-momentum tensor:

$$\begin{aligned} T_t^r = \frac{dr}{dr_*}T_t^{r_*} &\to \frac{1}{192\pi}[(e^{-2B(r)})'(e^{2A(r)})']_{r=r_+} + \frac{e^2}{\pi}A_u(r_+)^2 \\ &= \frac{1}{48\pi}\kappa_+^2 + \frac{e^2}{\pi}A_u(r_+)^2. \end{aligned} \tag{3.19}$$

The first term of (3.19) is derived from the Hawking flux and the second one is derived from electric-magnetic potential of black holes. We see that the flux of Hawking radiation is dominated by the surface gravity κ_+ in any cases.

In [10, 33], using the gravitational anomaly method, they also derive the Hawking flux of the general spherical black holes. We summarize their derivation in appendix A.

4 Higher Derivative Correction of the Hawking Flux

In this section, by using the method we established in the last section we calculate the Hawking flux from the four dimensional charged black hole which includes the higher derivative correction. We set the action [1] as follows

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + F_{\mu\nu}F^{\mu\nu} + aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right], \tag{4.1}$$

where G is the four dimensional Newton constant and a, b and c are infinitesimal parameters with the dimension $(mass)^2$. The corrected metric of the charged black hole is

$$ds^2 = e^{2A(r)}dt^2 - e^{2B(r)}dr^2 - r^2 d\Omega_2^2. \tag{4.2}$$

where up to order one of a, b and c

$$e^{2A(r)} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + (b+4c)\left(-\frac{2Q^2}{r^4} + \frac{2Q^2M}{r^5} - \frac{2Q^4}{5r^6}\right), \tag{4.3}$$

$$e^{-2B(r)} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + (b+4c)\left(-\frac{4Q^2}{r^4} + \frac{6Q^2M}{r^5} - \frac{12Q^4}{5r^6}\right), \tag{4.4}$$

M and Q is the mass and the charge of the uncorrected black hole. And the corrected electro-magnetic field is

$$A_t(r) = \frac{Q}{r} + \frac{Q^3}{5r^5}(b+4c). \tag{4.5}$$

The uncorrected black hole have a property

$$A(r) = -B(r), \tag{4.6}$$

however, this property is corrected. Up to order one of a, b and c, we have

$$e^{-2B(r)} = e^{2A(r)}L(r)^2,\ where\ L(r)^2 = 1 - \frac{2Q^2}{r^4}(b+4c). \tag{4.7}$$

The exact form of the $L(r)$ is shown in [1].

We know that the only quantity which is needed to obtain the Hawking flux is the surface gravity κ_+ (2.9). Now we can calculate the higher derivative correction to the Hawking flux. From (4.3) and (4.4), up to order one of a, b and c, the Hawking flux is

$$\frac{\kappa_+^2}{48\pi} = F_o + \delta F \tag{4.8}$$

where F_0 is an original Hawking flux

$$F_0 = \frac{(M^2 - Q^2 + M\sqrt{M^2 - Q^2})^2}{48\pi(M + \sqrt{M^2 - Q^2})^6} \tag{4.9}$$

and δF is the higher derivative correction of it

$$\delta F = (b+4c)\frac{20M^4Q^2 - 25M^2Q^4 + 6Q^6 + \sqrt{M^2 - Q^2}(20M^3Q^4 - 15MQ^6)}{240\pi(M + \sqrt{M^2 - Q^2})^{10}}. \tag{4.10}$$

5 Conclusion and Outlook

In this paper we obtain an explicit formula for the higher derivative corrected Hawking flux by the CFT method. As a by-product we obtain the general formula for any kind of spherical black holes in any kind of modified gravity by the CFT method. We found that the form (3.19) is the same as an ordinary black hole, and only depends on the surface gravity κ_+. We saw that tortoise coordinate r_* defined as (2.5) has no singularity in the region $r \geq r_+$. In Appendix A, we simplified the gravitational anomaly method given by [2] and many other papers.

As a future work, it is interesting to apply this method to non-spherical black holes. To find the explicit relationship between two methods [6] and [2] is also an interesting question remained.

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A Calculation of Corrected Hawking Flux via Gravitational Anomaly

In this appendix, we calculate the Hawking flux from four dimensional black hole with higher derivative correction by using the method studied by Robinson and Wilczek [2] (a modified derivation was shown by [3] and [35]). These papers are based on a gravitational anomaly of an effective chiral two dimensional theory. Because we treat with charged black hole, we must know $U(1)$ current of the two dimensional theory (as discussed in [3]). Since the theory is described as the two dimensional CFT in the near horizon region (Appendix B), we can separate ingoing modes and outgoing modes between the horizon r_+ and slightly outside $r_+ + \epsilon$. We will call the region "shell region". We decompose the two dimensional $U(1)$ current as

$$J^\mu(r) = J^\mu_{(o)}(r)\Theta_+(r) + (J^\mu_{(H)}(r) + K^\mu_{(H)}(r))H(r), \tag{A.1}$$

where $\Theta_+(r) = \Theta(r - \epsilon)$ and $H(r) = 1 - \Theta_+(r)$ ($\Theta(r) = 0, 1$ for $r < 0, r > 0$ respectively), these are step functions which take values in the shell region and

out the shell region respectively. $J^\mu_{(H)}(r)$ and $K^\mu_{(H)}(r)$ are respectively outgoing current and ingoing current. The current $J^\mu_{(H)}$ and $K^\mu_{(H)}$ obey conservation laws with consistent anomaly.

$$\nabla_\mu J^\mu_{(H)} = \frac{e^2}{4\pi\sqrt{-g}}\partial_r A_t \tag{A.2}$$

and

$$\nabla_\mu K^\mu_{(H)} = -\frac{e^2}{4\pi\sqrt{-g}}\partial_r A_t. \tag{A.3}$$

These equations can be integrated to

$$e^{A(r)+B(r)} J^r_{(H)}(r) = e^{A(r_+)+B(r_+)} c_H + \frac{e^2}{4\pi}(A_t(r) - A_t(r_+)), \tag{A.4}$$

and

$$e^{A(r)+B(r)} K^r_{(H)}(r) = -\frac{e^2}{4\pi}(A_t(r)), \tag{A.5}$$

where c_H is an integration constant. We set the integral constant of $K^r_{(H)}$ to zero, which condition means there is no ingoing mode from the infinity.

Outside the shell region, $J^\mu_{(o)}$ satisfies an ordinary conservation

$$\nabla_\mu J^\mu_{(o)} = 0, \tag{A.6}$$

and

$$e^{A(r)+B(r)} J^r_{(o)} = c_o, \tag{A.7}$$

where c_o is an integration constant which represents the value of the $U(1)$ current in infinity.

As [3] we introduce a new current

$$e^{A(r)+B(r)} \tilde{J}^\mu_{(H)} = e^{A(r)+B(r)} J^\mu_{(H)} + \frac{e^2}{4\pi} A_t(r), \tag{A.8}$$

$\tilde{J}^\mu_{(H)}$ satisfies the conservation law with covariant anomaly

$$\nabla_\mu \tilde{J}^\mu_{(H)} = \frac{e^2}{2\pi\sqrt{-g}}\partial_r A_t. \tag{A.9}$$

We require that $J^\mu(r)$ is smooth at the boundary of the shell and on the horizon the covariant current vanishes [2][3]. By these requirements we obtain

$$c_o = e^{A(r_+)+B(r_+)} c_H - \frac{e^2}{4\pi} A_t(r_+). \tag{A.10}$$

and

$$e^{A(r_+)+B(r_+)}c_H = -\frac{e^2}{4\pi}A_t(r_+). \tag{A.11}$$

From (A.10) and (A.11), we conclude

$$c_o = -\frac{e^2}{2\pi}A_t(r_+). \tag{A.12}$$

Next we calculate energy-momentum tensor. As we treat with J^μ we separate the energy-momentum tensor as

$$T^\mu_\nu(r) = T^\mu{}_{\nu(o)}(r)\Theta_+(r) + (T^\mu{}_{\nu(H)}(r) + S^\mu{}_{\nu(H)}(r))H(r), \tag{A.13}$$

where $T^\mu{}_{\nu(H)}$ is the outgoing current in the shell region and $S^\mu{}_{\nu(H)}(r)$ is the ingoing one there.

These currents satisfy the following conservation laws.

$$\begin{aligned}
\nabla_\mu T^\mu{}_{\nu(H)}(r) &= F_{\mu\nu}J^\mu_{(H)} + A_\nu(\nabla_\mu J^\mu_{(H)}) + \mathcal{A}_\nu && \text{(A.14)}\\
\nabla_\mu S^\mu{}_{\nu(H)}(r) &= F_{\mu\nu}K^\mu_{(H)} + A_\nu(\nabla_\mu K^\mu_{(H)}) - \mathcal{A}_\nu && \text{(A.15)}\\
\nabla_\mu T^\mu{}_{\nu(o)}(r) &= F_{\mu\nu}J^\mu_{(o)} + A_\nu(\nabla_\mu J^\mu_{(o)}) && \text{(A.16)}
\end{aligned}$$

where

$$\mathcal{A}_\nu = \frac{1}{96\pi\sqrt{-g}}\epsilon^{\beta\delta}\partial_\delta\partial_\alpha\Gamma^\alpha_{\nu\beta} \tag{A.17}$$

is the consistent gravitational anomaly. In the metric (4.2) we can calculate

$$\begin{aligned}
&e^{A(r)+B(r)}\mathcal{A}_r = 0, && \text{(A.18)}\\
&e^{A(r)+B(r)}\mathcal{A}_t = \partial_r N^r_t, && \text{(A.19)}\\
&\left(N^r_t = \frac{1}{192\pi}(e^{-2B(r)}(e^{2A(r)})')'\right). && \text{(A.20)}
\end{aligned}$$

Outside the shell region, from (A.16)

$$e^{A(r)+B(r)}T^r{}_{t(o)} = a_o + c_o A_t(r), \tag{A.21}$$

where a_o is the integration constant which means the energy momentum flux at infinity.

In the shell region, from (A.16) we have

$$\begin{aligned}
e^{A(r)+B(r)}T^r{}_{t(H)} &= a_H + \left[c_o A_t(r) + \frac{e^2}{4\pi}A_t^2 + N^r_t\right]^r_{r_H}, && \text{(A.22)}\\
e^{A(r)+B(r)}K^r{}_{t(H)} &= -\left(c_o A_t(r) + \frac{e^2}{4\pi}A_t^2 + N^r_t\right), && \text{(A.23)}
\end{aligned}$$

where a_H is an integration constant, and we again set the integral constant of $K^r{}_t$ to zero.

From the smoothness of the T^μ_ν, we have

$$a_o = a_H + \frac{e^2}{4\pi} A_t(r_+)^2 - N^r_t(r_+). \tag{A.24}$$

To obtain a_o, we have to know a_H. We take the similar argument of evaluating of c_H.

We introduce a new energy momentum tensor:

$$e^{A(r)+B(r)}\tilde{T}^r{}_t = e^{A(r)+B(r)} T^r{}_t + \frac{1}{96\pi} e^{2A(r)-2B(r)} (A^{''}(r) - 2A^{'}(r)^2). \tag{A.25}$$

The new energy momentum tensor satisfies a conservation law with a covariant anomaly,

$$\nabla_\mu \tilde{T}^\mu{}_{\nu(H)}(r) = F_{\mu\nu} J^\mu_{(H)} + A_\nu (\nabla_\mu J^\mu_{(H)}) + \tilde{\mathcal{A}}_\nu, \tag{A.26}$$

where

$$\tilde{\mathcal{A}}_\nu \equiv \frac{1}{96\pi\sqrt{-g}} \epsilon_{\mu\nu} \partial^\mu R \tag{A.27}$$

Now we require that the covariant flux must vanish at the horizon. This requirement leads to

$$a_H = 2N^r_t(r_+), \tag{A.28}$$

and from (A.24)

$$a_o = \frac{e^2}{4\pi} A_t(r_+)^2 + N^r_t(r_+). \tag{A.29}$$

Using of the surface gravity κ_+ (2.9),

$$a_o = \frac{e^2}{4\pi} A_t(r_+)^2 + \frac{\kappa_+^2}{48\pi}. \tag{A.30}$$

This is the Hawking flux at infinity and is the same one as (3.19)

B The Emergence of the Two dimensional CFT Near the Horizon

Now we describe the reason of having a property of two dimensional CFT near the horizon. For simplicity, we treat with a real scalar field ϕ on the general four dimensional spherical symmetric black hole spacetime which metric is given by (2.1).

First, we see the kinetic term describes an effective two dimensional theory near the horizon. The kinetic term is given by

$$\begin{aligned} I_k &= \int d^4x\sqrt{-g}\ \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \\ &= \int dtdrd\theta d\varphi\ r^2\sin\theta\left(\frac{1}{2}e^{A(r)+B(r)}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right). \end{aligned} \tag{B.1}$$

Under a transformation $r \to r_*$ (r_* is defined by (2.5)), this becomes

$$I_k = \int dtdr_*d\theta d\varphi\ r^2\sin\theta\frac{1}{2}\left[(\partial_t\phi)^2 - (\partial_{r_*}\phi)^2 - e^{2A(r)}r^{-2}\left((\partial_\theta\phi)^2 + (\partial_\varphi\phi)^2\right)\right]. \tag{B.2}$$

Thus, as $r \to r_+$, only first two terms survive and I_k would describe a two dimensional theory. More explicitly, a partial wave expansion

$$\phi = \sum_{l,m}\phi_{lm}(t,r_*)Y_{lm}(\theta,\varphi), \tag{B.3}$$

where $Y_{lm}(\theta,\varphi)$ are spherical harmonics, yields

$$I_k = \int dtdr_*r^2\sum_{l,m}\frac{1}{2}[|\partial_t\phi_{lm}(t,r_*)|^2 - |\partial_{r_*}\phi_{lm}(t,r_*)|^2], \tag{B.4}$$

near the horizon. So the theory becomes a two dimensional theory with dilaton.

Next, we show vanishing of the mass term and interaction terms. It is very easy to show. We consider ϕ^n ($n = 2, 3, \dots$) terms, and

$$\int dtdrd\theta d\varphi\ \phi^n = \int dtdr_*d\theta d\varphi\ r^2\sin\theta L(r)e^{2A(r)}\phi^n. \tag{B.5}$$

So the mass term and interaction terms vanish near the horizon.

Thus we conclude that near the horizon the theory becomes an effective two dimensional CFT.

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